

## Sections 13.5 and 13.6

Review of Right  $\Delta$  Trig (Fri)

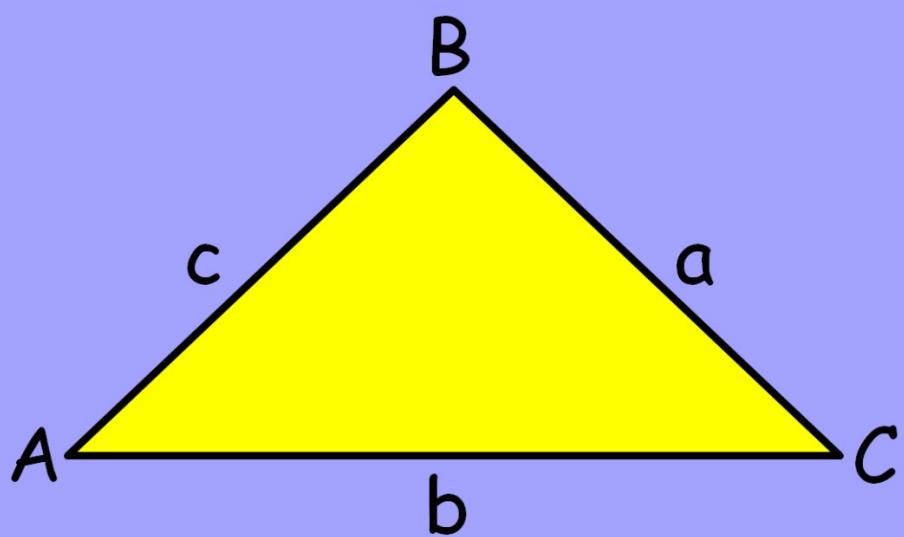
Law of Sines (Mon & Tues)

Law of Cosines (Weds & Thurs)

Objective:

To use trigonometry to solve oblique (non-right) triangles.

## Standard Notation for a Triangle



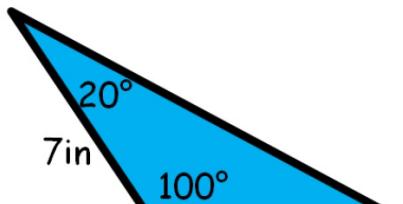
A, B, and C refer to the angles of the triangle.  
a, b, and c refer to the side lengths of the triangle.

To be able to solve a triangle\* you need to know...

- 1) One side length.
- 2) Any other 2 pieces of information (for example 1 additional side length and 1 angle measure.)

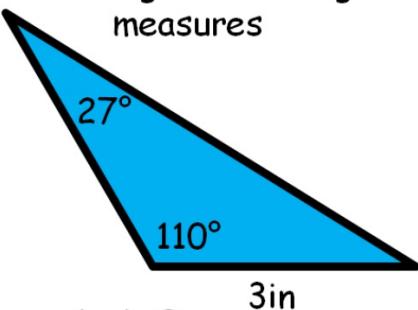
"to solve a triangle" means that you find all three side lengths and all three angle measures.

1 side length and 2 angle measures



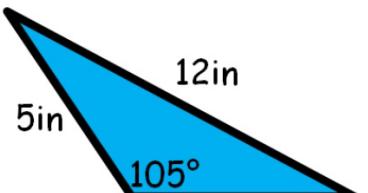
**ASA**

1 side length and 2 angle measures



**AAS**

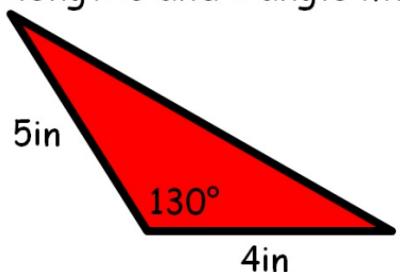
2 side lengths and 1 angle measure



**SSA**

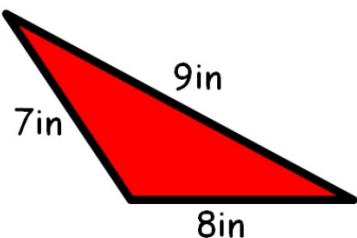
## LAW OF SINES

2 side lengths and 1 angle measure



**SAS**

3 side lengths



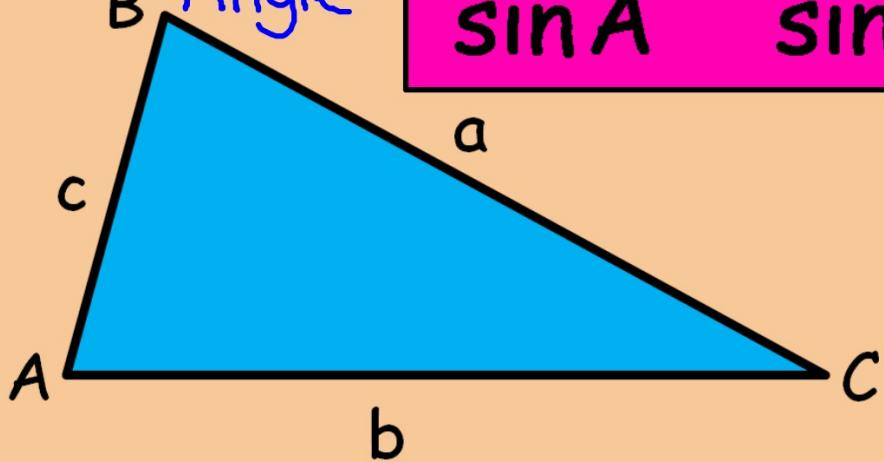
**SSS**

## LAW OF COSINES

## Law of Sines -

If  $\triangle ABC$  is a triangle with sides  $a$ ,  $b$ , and  $c$ , then

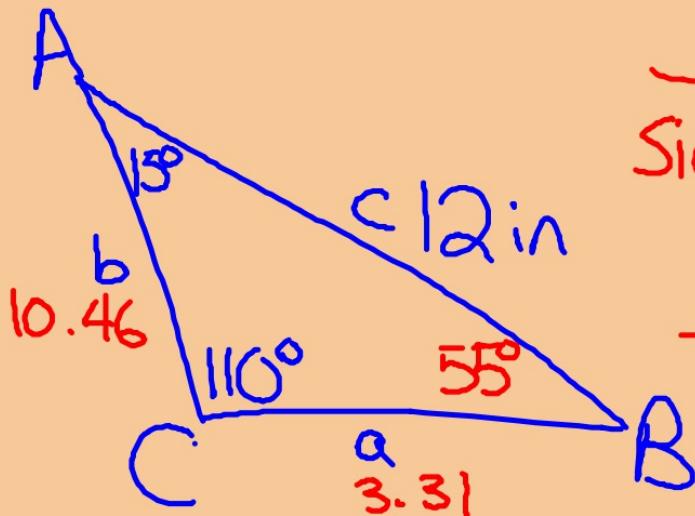
Side  
Angle



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Example 1: *Solve*

$$A = \underline{15^\circ}, C = \underline{110^\circ}, c = 12\text{ in}$$



$$\frac{12}{\sin(110)} = \frac{a}{\sin(15)}$$

$$\frac{12 \sin(15)}{\sin(110)} = \frac{a \sin(110)}{\sin(110)}$$
$$3.31 = a$$

$$\frac{12}{\sin(110)} = \frac{b}{\sin(55)}$$

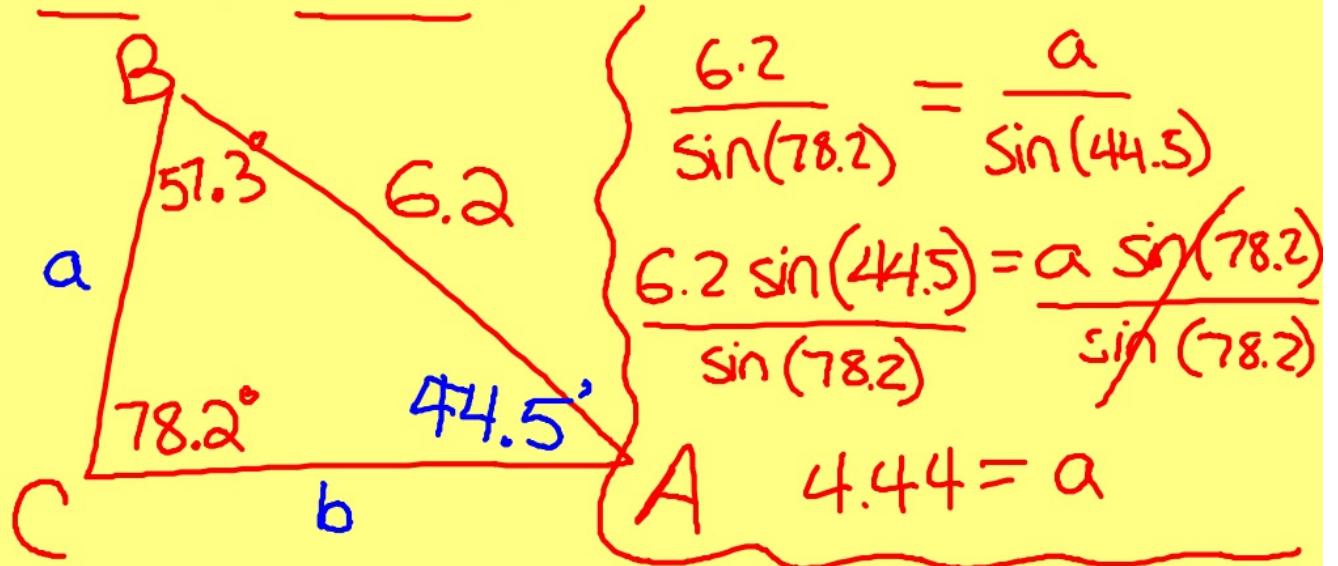
$$\frac{12 \sin(55)}{\sin(110)} = \frac{b \sin(110)}{\sin(110)}$$

Does it  
make  
sense?!

$B = \underline{55^\circ}$
$a = \underline{3.31\text{ in}}$
$b = \underline{10.46\text{ in}}$

Example 2 -

$$B = \underline{57.3^\circ}, C = \underline{78.2^\circ}, c = 6.2\text{cm}$$



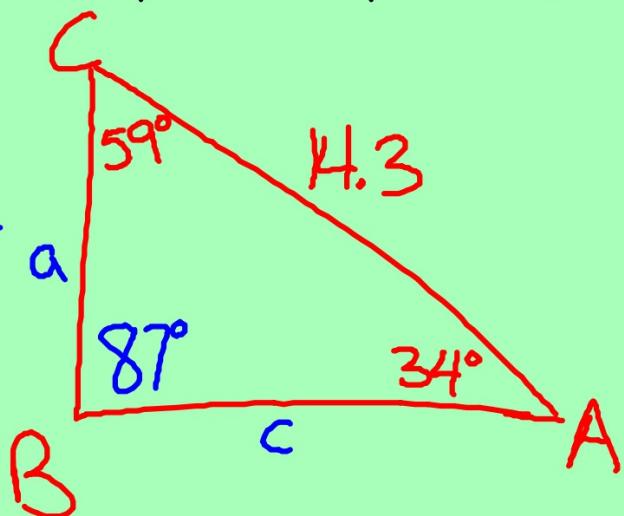
$$\frac{6.2}{\sin(78.2)} = \frac{b}{\sin 57.3}$$

Does it make sense?!

$A = 44.5^\circ$
$a = 4.44\text{cm}$
$b = 5.33\text{cm}$

Example 3 -

$$A = 34^\circ, C = 59^\circ, b = 14.3 \text{ cm}$$



$$\begin{aligned} \frac{14.3}{\sin(87)} &= \frac{a}{\sin(34)} \\ 14.3 \sin(34) &= a \frac{\sin(87)}{\sin(34)} \\ 8.01 &= a \end{aligned}$$

$$\frac{14.3}{\sin 87} = \frac{c}{\sin 59} \quad c = 12.27$$

$B = 87^\circ$
$a = 8.01$
$c = 12.27$

Example 4 -

$$A = 28^\circ, B = 62^\circ, b = 19\text{cm}$$

$$\begin{aligned}C &= \underline{90^\circ} \\a &= \underline{10.10\text{cm}} \\c &= \underline{21.52\text{cm}}\end{aligned}$$

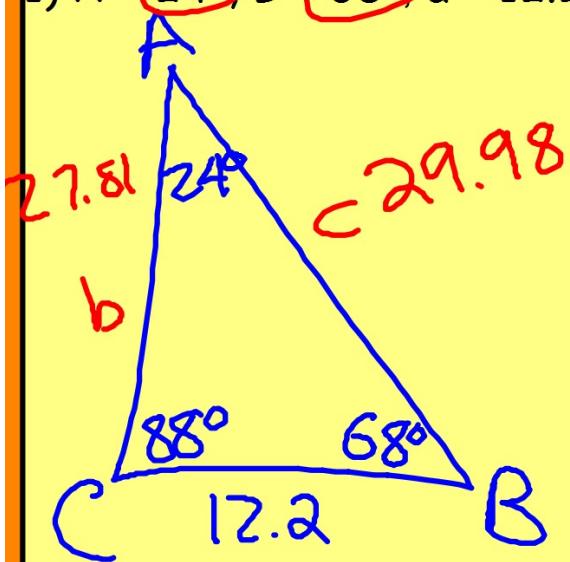
## 1/12 HW Answers

- ①  $A = 36^\circ$   $a = 14.72$   $c = 23.53$
- ②  $C = 46$   $a = 6.53$   $b = 6.25$
- ③  $B = 43^\circ$   $b = 6.66$   $c = 9.67$
- ④  $C = 75$   $a = 24.89$   $b = 30.48$
- ⑤  $A = 49^\circ$   $a = 41.80$   $c = 53.96$
- ⑥  $A = 26^\circ$   $a = 10.44$   $b = 21.03$

End of 1/12 Notes

Warm Up - solve for the missing sides and/or angles

1)  $A = 24^\circ$ ,  $B = 68^\circ$ ,  $a = 12.2\text{ft}$



$$c = \frac{88}{\sin(24)}$$

$$b = \frac{27.81}{\sin(24)}$$

$$c = \underline{\underline{29.98}}$$

2)  $B = 104^\circ$ ,  $C = 33^\circ$ ,  $a = 18.1$

$$\frac{12.2}{\sin(24)} = \frac{b}{\sin(68)}$$

$$\frac{12.2 \sin 68}{\sin(24)} = \frac{b \sin(68)}{\sin(24)}$$

$$27.81 = b$$

$$A = \underline{\underline{\hspace{2cm}}}$$

$$b = \underline{\underline{\hspace{2cm}}}$$

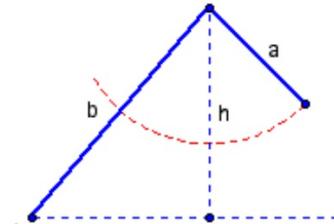
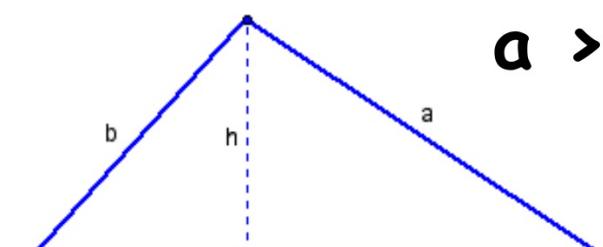
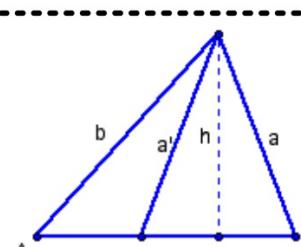
$$c = \underline{\underline{\hspace{2cm}}}$$

## The Ambiguous Case: SSA

- No Solution (No such triangle exists)
- One Solution (One triangle exists)
- Two Solutions (Two triangles satisfy the conditions)

**REMEMBER:**

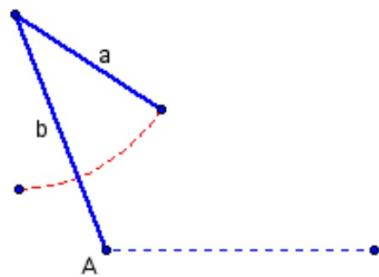
If it spells out SSA, then it is  
a pain in the...

	Diagram	Triangles Possible
Given angle is acute	<p>If <math>a &lt; b</math> but <math>a &lt; h</math></p> 	None
	<p><math>a &gt; b</math></p> 	One
	<p><math>h &lt; a &lt; b</math></p> 	Two

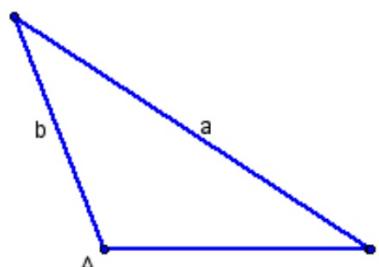
## Diagram      Triangles Possible

Given  
angle is  
obtuse

$\angle A$  is  
obtuse



$$a \leq b$$



$$a > b$$

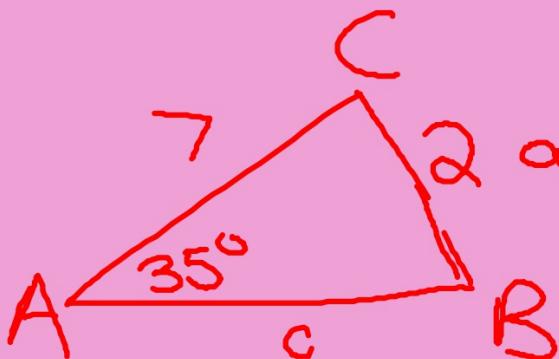
None

One

Example 1: SSA

$$A = 35^\circ, b = 7 \text{ in}, a = 2 \text{ in}$$

0 or 2  
solutions



$$\frac{2}{\sin(35)} \neq \frac{7}{\sin(B)}$$

$$2 \sin(B) = 7 \sin 35$$

$$\sin B = \frac{7 \sin(35)}{2}$$
$$B = \sin^{-1} \left( \frac{7 \sin(35)}{2} \right)$$

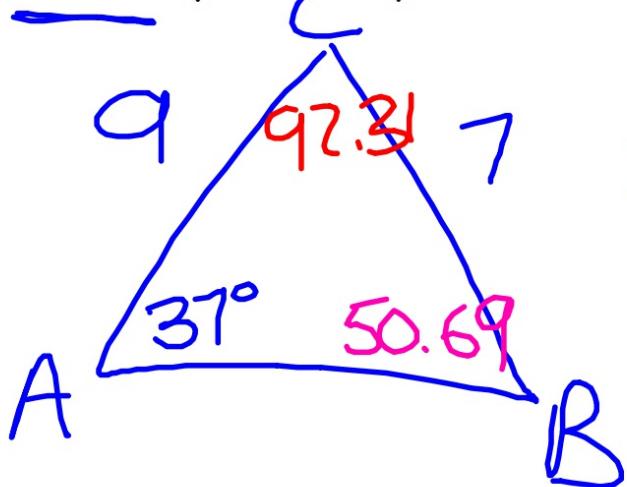
~~B =~~  
~~C =~~  
~~c =~~  
NO triangle

Example 2:

SSA

$$\underline{A = 37^\circ}, b = 9 \text{ in}, a = 7 \text{ in}$$

0 or 2



$$\frac{7}{\sin(37)} = \frac{9}{\sin(B)}$$

$$7 \sin(B) = 9 \sin(37)$$

$$\sin(B) = \frac{9 \sin(37)}{7}$$

$$B = \sin^{-1}\left(\frac{a \sin(37)}{7}\right)$$

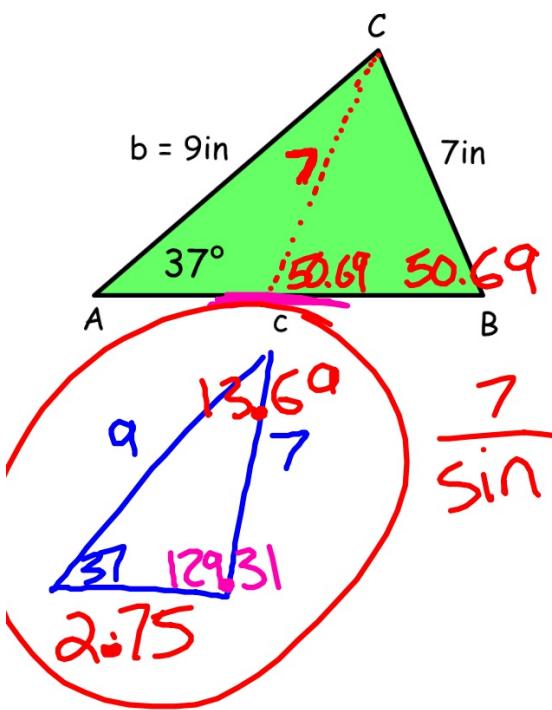
$$B = \frac{50.69}{7}$$

$$C = \frac{92.31}{7}$$

$$c = \underline{11.62}$$

$$\frac{7}{\sin(37)} = \frac{c}{\sin(92.31)}$$

$$\frac{c \cdot \sin(37)}{\sin(37)} = \frac{7 \sin(92.31)}{\sin(37)}$$



$$B = \underline{129.31^\circ}$$

$$C = \underline{13.69^\circ}$$

$$c = \underline{2.75}$$

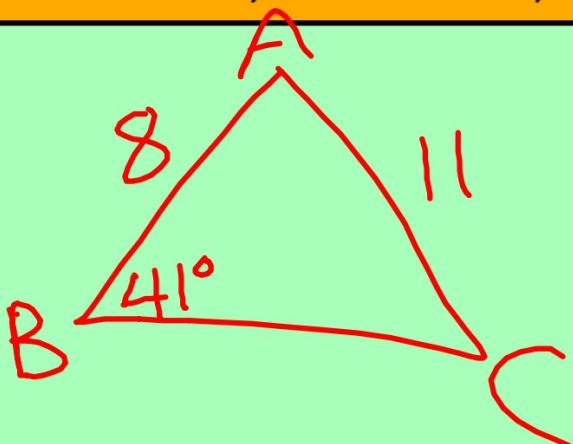
$$\frac{7}{\sin(37)} = \frac{c}{\sin(13.69)}$$

This is NOT a unique triangle. This is why SSA does NOT always work as a congruence theorem in Geometry!

Example 3: SSA

$B = 41^\circ$ ,  $b = 11\text{cm}$ ,  $c = 8\text{cm}$

(1)



$A = \underline{\underline{110.50}}$
$C = \underline{\underline{28.50}}$
$a = \underline{\underline{15.70}}$

Example 4: \_\_\_\_\_

$A = 102^\circ$ ,  $b = 5\text{in}$ ,  $a = 8\text{in}$

**B** = \_\_\_\_\_  
**C** = \_\_\_\_\_  
**c** = \_\_\_\_\_

**Example 5:** \_\_\_\_\_

$A = 120^\circ$ ,  $a = 9\text{in}$ ,  $c = 4\text{in}$

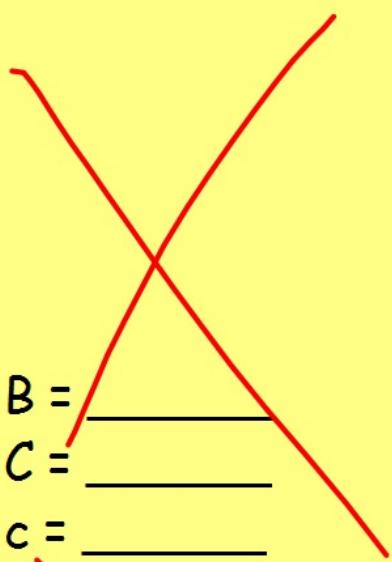
**B** = \_\_\_\_\_  
**C** = \_\_\_\_\_  
**b** = \_\_\_\_\_

## Homework

1.  $B = 130^\circ$   $a = 10$   $b = 8$
2.  $A = 20^\circ$   $a = 10$   $c = 11$
3.  $C = 95^\circ$   $a = 8$   $c = 9$
4.  $A = 70^\circ$   $B = 60^\circ$   $c = 25$
5.  $C = 16^\circ$   $b = 92$   $c = 32$
6.  $A = 10^\circ$   $C = 130^\circ$   $b = 5$

## Warm Up - solve for the missing sides and/or angles

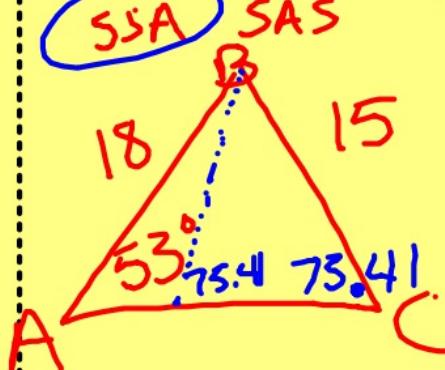
1)  $A = 76^\circ$ ,  $a = 34$ ,  $b = 21$



$$\begin{aligned}B &= \underline{\hspace{2cm}} \\C &= \underline{\hspace{2cm}} \\c &= \underline{\hspace{2cm}}\end{aligned}$$

~~Other set of solutions?~~

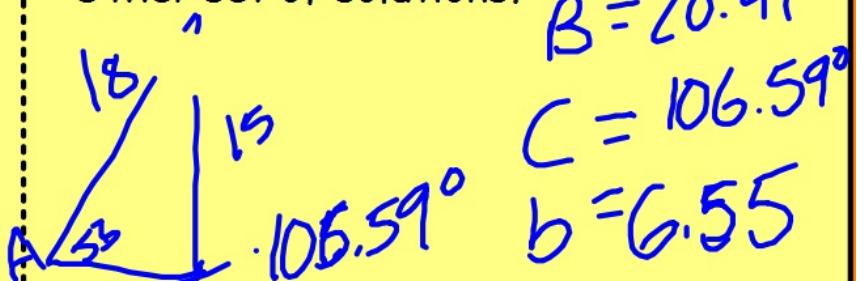
2)  $A = 53^\circ$ ,  $a = 15$ ,  $c = 18$



$\bigcirc$  or  $\bigcirc 2$

$$\begin{aligned}B &= \underline{53.59^\circ} \\C &= \underline{73.41^\circ} \\b &= \underline{15.12}\end{aligned}$$

~~Other set of solutions?~~



## **HOMEWORK: Solutions**

**page 804 #29-34**

**1) SSA No Solution**

**2) SSA Two Solutions**

$$B = 137.90^\circ \quad C = 22.10^\circ \quad b = 19.60$$

$$B = 2.1^\circ \quad C = 157.9^\circ \quad b = 1.07$$

**3) SSA One Solution**

$$A = 62.31^\circ \quad B = 22.69^\circ \quad b = 3.48$$

**4) ASA One Solution!**

$$C = 50^\circ \quad a = 30.67 \quad c = 28.26$$

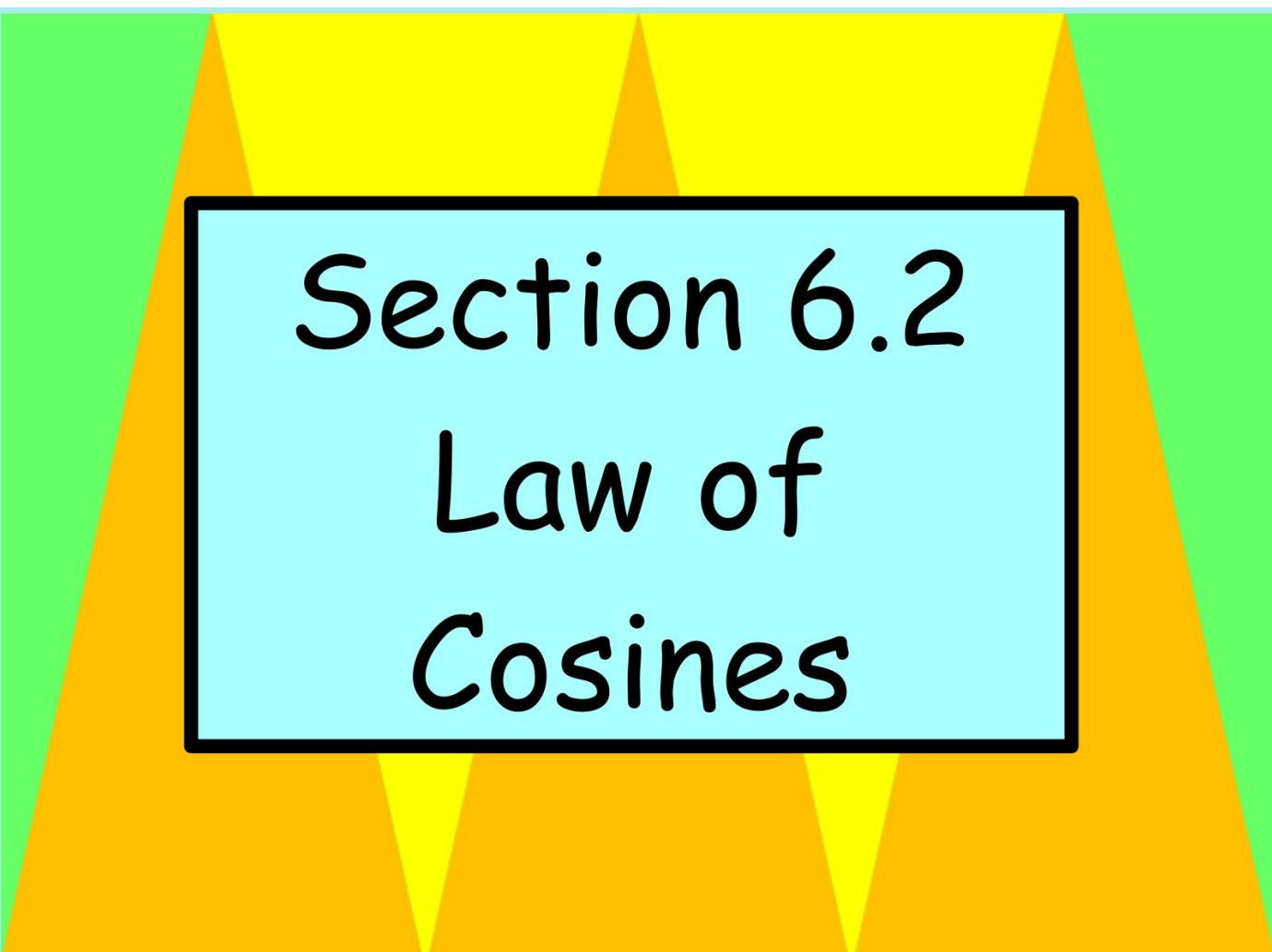
**5) SSA Two Solutions!**

$$A = 111.58^\circ \quad B = 52.42^\circ \quad a = 107.96$$

$$A = 36.42^\circ \quad B = 127.58^\circ \quad a = 68.93$$

**6) ASA One Solution**

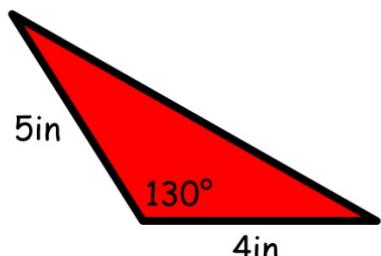
$$B = 40^\circ \quad a = 1.35 \quad c = 5.96$$



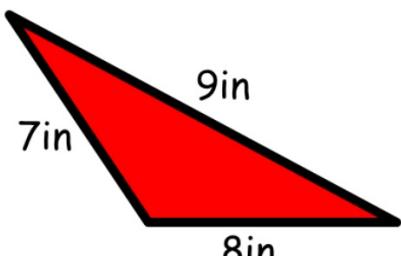
# **Section 6.2**

## **Law of Cosines**

- Used to solve triangles that are not right triangles
- Used when there is no corresponding angle and side given.



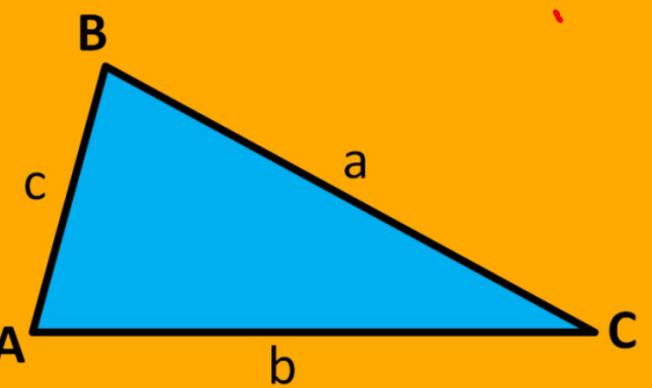
**SAS**



**SSS**

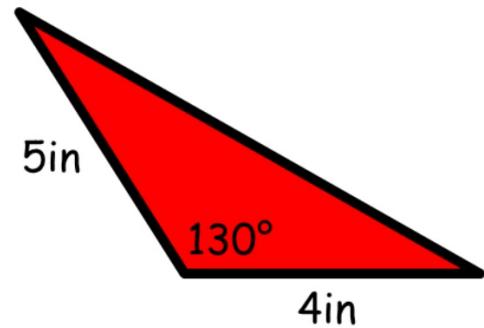
## LAW OF COSINES:

$$a^2 = b^2 + c^2 - (2bc)\cos A$$



## Law of Cosines

### CASE 1: SAS

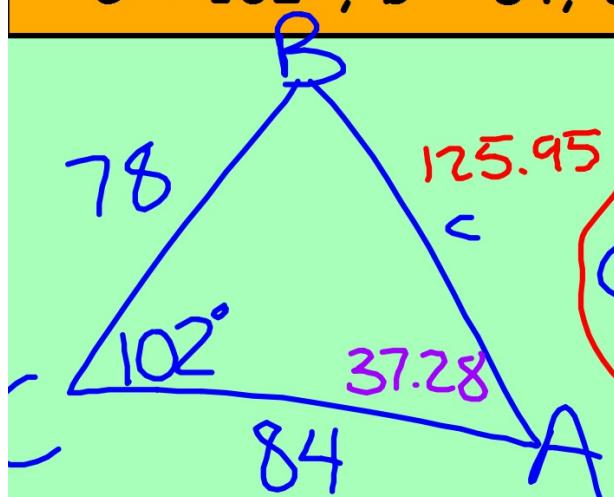


- 1.) Fill givens into the Law of Cosines option that corresponds to the given angle. Find the missing side length.
  
- 2.) Use Law of Sines to find the smallest missing angle measure.
  
- 3.) Subtract from 180 to find the last missing angle measure.



Example 1: SAS

$$C = 102^\circ, b = 84, a = 78$$



$$c^2 = a^2 + b^2 - 2(a \cdot b) \cos(C)$$

$$c^2 = 78^2 + 84^2 - 2 \cdot 78 \cdot 84 \cos(102^\circ)$$

$$c^2 = 15864.47$$

$$c = 125.95$$

$$\frac{125.95}{\sin(102^\circ)} = \frac{78}{\sin(A)}$$
$$A = \sin^{-1} \left( \frac{78 \sin(102^\circ)}{125.95} \right)$$

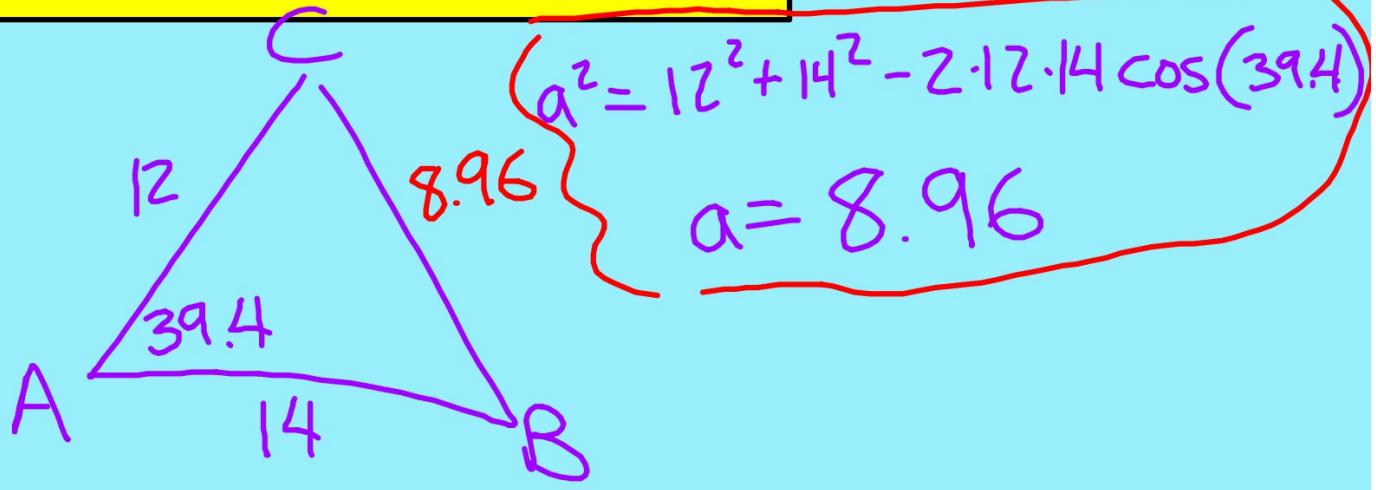
$$A = \underline{\underline{37.28^\circ}}$$

$$B = \underline{\underline{40.72^\circ}}$$

$$C = \underline{\underline{125.95}}$$

Example 2: SAS

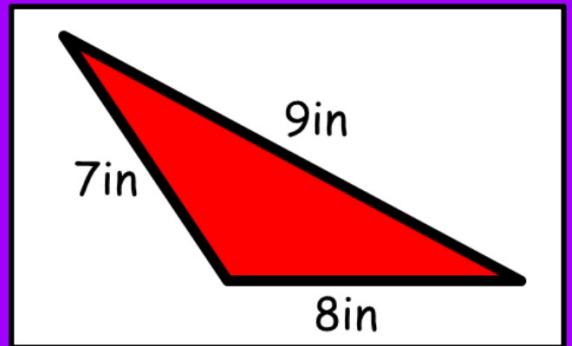
$$A = 39.4^\circ, c = 14, b = 12$$



B =	<u>58.22°</u>
C =	<u>82.38°</u>
a =	<u>8.96</u>

## Law of Cosines

### CASE 2: SSS

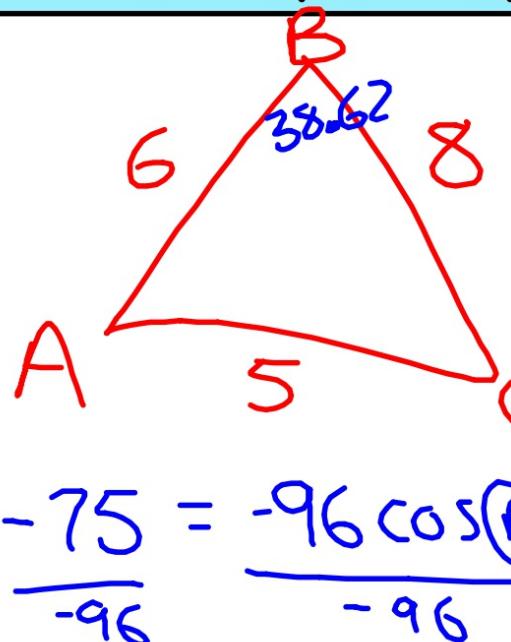


- 1.) Fill into the three Law of Cosines Options to get the smallest angle measure.
  
- 2.) Use Law of Sines to find the next smallest missing angle measure.
  
- 3.) Subtract from 180 to find the largest angle measure.



Example 3: SSS

$$a = 8, b = 5, c = 6$$



$$5^2 = 6^2 + 8^2 - 2(6)(8)\cos(B)$$

$$25 = 36 + 64 - 96\cos(B)$$

$$25 = 100 - 96\cos(B)$$

$$\frac{-75}{-96} = \frac{-96\cos(B)}{-96}$$

$$\frac{75}{96} = \cos(B)$$

$$B = \cos^{-1}\left(\frac{75}{96}\right)$$

A =	<u>92.88°</u>
B =	<u>38.62°</u>
C =	<u>48.50°</u>

**Example 4:** a = 19, b = 24.3, c = 21.8

$$\begin{aligned}A &= \underline{\underline{48.30^\circ}} \\B &= \underline{\underline{72.76^\circ}} \\C &= \underline{\underline{58.94^\circ}}\end{aligned}$$

## HW #34

- 1)  $B = 20^\circ$   $a = 120$   $c = 100$
- 2)  $C = 95^\circ$   $a = 10$   $b = 12$
- 3)  $a = 25$   $b = 11$   $c = 24$
- 4)  $a = 2$   $b = 4$   $c = 5$
- 5)  $a = 5$   $C = 24^\circ$   $b = 8$
- 6)  $a = 32$   $b = 39$   $c = 16$

## **HOMEWORK: Solutions**

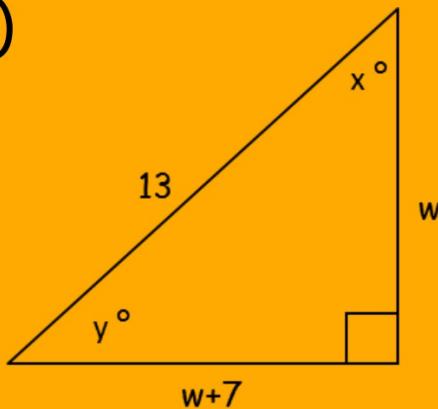
page 810 #5-8, 15-17

- 1) SAS       $A = 107.4^\circ$     $C = 52.7^\circ$     $b = 43.0$**
- 2) SAS       $A = 37.7^\circ$     $B = 47.3^\circ$     $c = 16.3$**
- 3) SSS       $A = 82.2^\circ$     $B = 25.8^\circ$     $C = 72.0^\circ$**
- 4) SSS       $A = 22.3^\circ$     $B = 49.5^\circ$     $C = 108.2^\circ$**
- 5.) SAS       $A = 35.2^\circ$     $B = 112.8^\circ$     $a = 4.60$**
- 6.) SSS       $A = 52.9^\circ$     $B = 103.6^\circ$     $C = 23.5^\circ$**

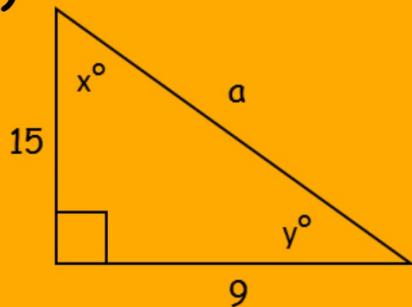
## Trig and Special Right Triangle Review

-Solve for all missing sides and angles-

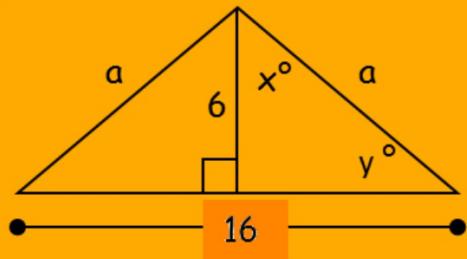
1)



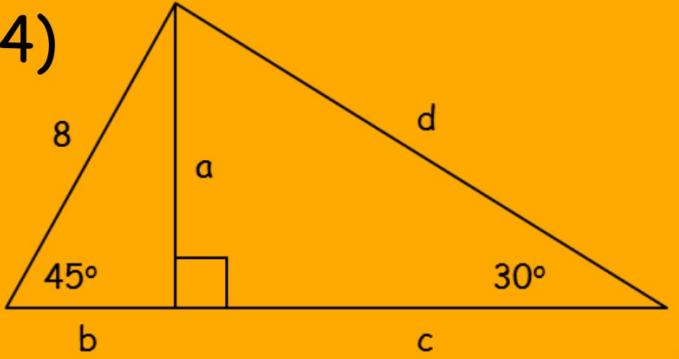
2)



3)



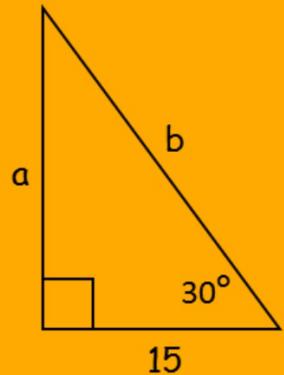
4)



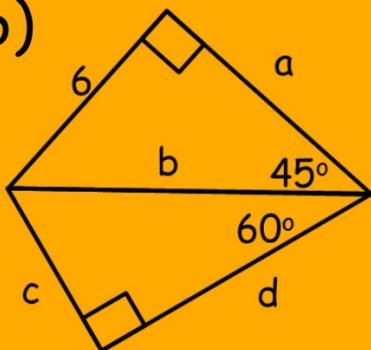
## Trig and Special Right Triangle Review

-Solve for all missing sides and angles-

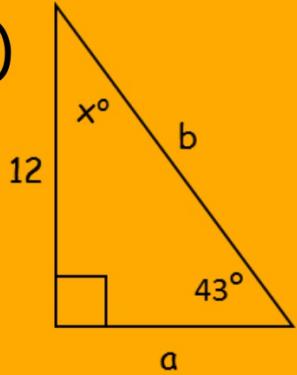
5)



6)



7)



8)

